Three-class triple junction inpainting

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Abbreviation:	inpainting-n4/inpainting-n8
Number of instances:	2
Number of variables:	$\sim 1.4 \times 10^4$ (120×120)
Number of labels:	3+1 for boundary
Number of factors:	$\sim 4 \times 10^4$ (n4), $\sim 7 \times 10^4$ (n8)
Order:	2
Function type:	Potts

Description Three-class inpainting in circular domain, modified from [2]. The data term is given only at the boundary, i.e., inside the ring. The problem is extended to a rectangular domain by introducing a fourth label for the outside pixels to make it easily accessible for solvers that rely on a rectangular grid. The analytical solution is unique and has a single triple junction at the center where the three interfaces meet at 120 degree angle.

The problem set also contains the "inverted" problem where the signs of the unary potentials haven been flipped. This model is difficult for solvers that rely on convex relaxation, since it permits (in the continuous formulation) arbitrarily many globally optimal solutions. For every solution an equally good but different solution can be obtained by permuting the labels.

Figure 1: Three-class inpainting problem. Left to right: input with inpainting region marked in black, exemplary result, input corresponding to the "inverse" variant, exemplary result. The "inverse" variant has many equally good minimizers.

Objective / Learning The objective function is

$$J(x) = \sum_{v \in V} \varphi_i(x_i) + \sum_{ij \in E} \varphi_{ij}(x_i, x_j). \tag{1}$$

which discretizes the continuous functional

$$J(u) = \int_D \|c_{u(x)} - I(x)\|_2 dx + \lambda \mathcal{L}(u), \tag{2}$$

where $u : \Omega \to \{0, 1, 2\}$ is the label function, $\mathcal{L}(u)$ is the total boundary length, and $D \subseteq \Omega$ is the inpainting domain, and we additionally enforce $u(x) = I(x)$ outside on $\Omega \setminus D$. The regularization weight λ was set manually.

The Potts regularizer has been implemented using pairwise potentials with 4-neighborhoods (-n4) and 8-neighborhoods (-n8) with the pairwise factor weight chosen optimally according to [1].

References

[1] Y. Boykov. Computing geodesics and minimal surfaces via graph cuts. In *ICCV*, 2003.

[2] A. Chambolle, D. Cremers, and T. Pock. A convex approach to minimal partitions. *J. Imaging Sci.*, 5(4):1113–1158, 2012.